Modeling Ballistic Impact into Flexible Materials

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A versatile model for ballistic impact into flexible materials is presented along with corroborative experimental data. The dynamic problem is approximated by a time series of static analysis problems. An algorithm is described wherein in each time step the material properties of the target and the geometry of the deflection cone are derived, allowing complete description of the impact. Simple laboratory measurements of one impact condition, when incorporated into the algorithm, allow prediction of the results of a variety of other impact conditions. An experimental program was performed involving 0.22 and 0.357 caliber bullets fired at Kevlar 29 fabric (single- and multiple-layer) targets. Results compare favorably with the theory.

Nomenclature

a	= acceleration placed upon bullet by fabric, m/s/s
C	= speed of propagation of sound in the fabric, m/s
E	= Young's modulus of the fabric, N/m^2
h	= thickness of the fabric found by h = weight per area/ρ, cm
m	= mass of projectile, g
$R1$	= radius of the top of the conical shell defined by the circle of intersection of the projectile and the fabric, cm
$R2$	= radius of the base of the conical shell defined by the position of the transverse wave front, cm
t	= time, s
$UBAR$	= speed of transverse wave in the fabric, m/s
$Utot$	= height of the conical shell, cm
V_{str}	= velocity of projectile striking target, m/s
V_{res}	= velocity of projectile leaving target, m/s
y	= axial position from tip of bullet, cm
β	= contact angle defined by included angle between centerline and side of conical shell, deg
ϵ	= strain, cm/cm
ρ	= mass density of the fabric, g/cc

I. Introduction

PREDICTION of a material's behavior during a high-speed impact is a challenging task. Extrapolation of low strain rate test data to the ballistic realm is misleading due to the sensitivity of material properties to strain rate. In woven flexible materials, additional factors involving fiber-fiber interaction and straightening of the undulations in the weave can add to the complexity of the problem. For these reasons, an investigation of a new material mandates experimentation at all impact conditions of interest, with little latitude for extrapolation. The typical investigation results in a plot that, we will see, is quite limited in scope. We present a versatile investigative tool that provides information on the geometry during impact and also allows extrapolation of ballistic impact data.

The common means of presenting impact data is the $V_{res} - V_{str}$ plot, obtained by simply plotting the residual velocity (velocity after penetration) vs the striking velocity. Figure 1 shows how at high velocities the fabric target barely decelerates the bullet, whereas at lower velocities, the bullet is arrested. The experimentalist might estimate that a striking velocity of less than 250 m/s would result in arrest. However, that result is only valid for that striking condition. If a different bullet were used, or if one were to change the number of layers of fabric in the target, the results would change. A complete characterization, therefore, would require a different plot for each possible striking condition. Another weakness of this type of plot is its failure to predict the impact geometry. An impact that results in an arrest but deflects significantly in order to do so would nonetheless be harmful to the underlying contents. These limitations of the $V_{res} - V_{str}$ plot encourage improvements in impact analysis.

In 1975, Vinson and Zukas[1] developed an algorithm to model transverse impact by treating the fabric as an homogeneous, isotropic, elastic plate, which deforms into the shape of a straight-sided conical shell. This allows for easy analysis of the loading on the bullet and, hence, its deceleration at any time. Successive time steps allow estimations of the interaction (including geometry, velocities, and strain distributions) from first contact until either the projectile is arrested or penetration occurs with some residual velocity. By inserting into the algorithm the striking and residual velocity data from an actual shot, one can identify a failure strain for the fabric and use this result to predict the outcome of a variety of impact conditions, including bullet geometry and multiple-layer targets with variable spacing. All of this is possible even though the only knowledge of the fabric is its density and weight per area.

We now present a more comprehensive description of the theory, some interesting details on the variables involved, and the results of a large experimental program which generally appears to corroborate the model.

II. Theory and Algorithm

The Vinson-Zukas model ignores the microscopic (strain energies) and macroscopic (yarn size, crossovers) approaches to the problem and simply views the fabric as a very flexible plate. One can imagine looking at a piece of fabric, noting the individual yarns, perhaps even being able to see the fibers in the yarns and the undulations of the yarns over their neighbors. Now, still looking at the fabric, one can let it slip out of focus. The details disappear, and the fabric now looks like an

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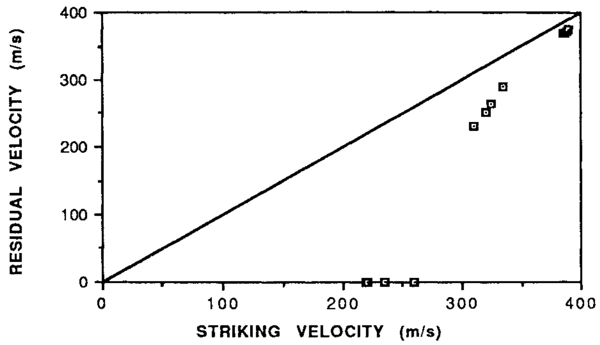

Fig. 1 The $V_{str} - V_{res}$ plot: the common method of presenting impact data. Data are for layers of Kevlar being impacted by a 0.357 caliber bullet.

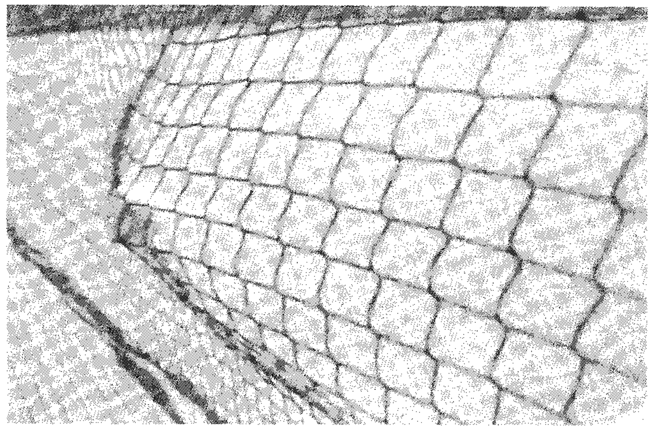

Fig. 2 Analogy of bullet/fabric to ball/net.

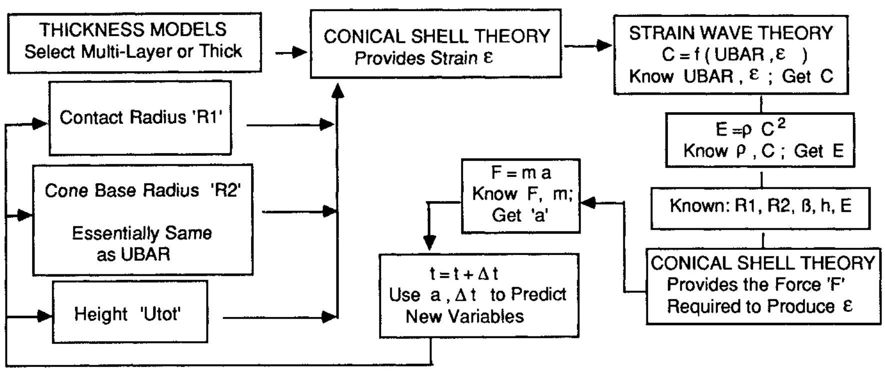

Fig. 3 The algorithm.

isotropic, homogeneous plate. All that is known about this plate is its density and weight per area and that it can elastically strain considerably more than the typical (stiff metal) plate one encounters.

A helpful analogy would be a tennis net. Left alone, it is a vertical plate with unknown properties. All that is known about it is that when impacted by a tennis ball, it deflects elastically into roughly a conical shape and arrests the ball. Comparably, the "net" in this problem is the fabric and the "ball" is the bullet (see Fig. 2). During this work, it is important to remember that we work with the properties of this assumed isotropic fabric, not the individual fibers. Equating the properties of the fabric target to the properties of a single fiber is as invalid as equating the properties of the tennis net to the properties of the string comprising it. The tennis net has vastly different properties than the individual strings.

The present approach models this dynamic problem as a series of quasistatic snapshots. Based on the mass and striking velocity of the bullet and the two known properties of the fabric, the geometry of the deformation cone of the fabric is approximated at each time step. The problem at hand, then, is that of a conical shell being axially loaded at its tip. The question to be answered is, "What is the loading involved which has caused such a deformation?" A problem of this type is easily handled by a structural theory for conical shells.[2] The variables that enter into such an analysis are those describing the geometry as well as the material properties. While these variables are normally well known in a static structural analysis problem, here they are not well defined. They must be approximated. Complete descriptions of the variables and how they are obtained is given in Sec. III.

Once the input variables are known, the situation is that of a plate of estimated physical properties deformed into a conical shape with a predictable strain distribution. Incorporating a relationship from wave theory allows for calculation of the instantaneous modulus of the material. Combining the known strain distribution with this modulus provides a stress distribution. The conical shell theory then allows one to calculate the load applied by the bullet to the fabric required to cause such a stress. By Newton's third law (action-reaction), this is also the load (force) applied by the fabric onto the bullet. Incorporating the mass of the bullet allows calculation ($F = ma$) of the deceleration being placed upon the bullet by the fabric.

At this point we have progressed to estimations of all facets of the interaction: the strain distribution, the geometry, positions, velocities, and, most important, the (de)acceleration applied to the bullet. Incrementing the time allows prediction of the geometry, positions, and velocities for the next time step. Figure 3 shows the algorithm. Through successive time steps, the model can continue to describe the impact until the velocity is brought to zero.

Figure 4 shows a plot of strain and velocity. If the actual shot resulted in arrest, the maximum strain that the fabric withstood could be identified. For the case shown in Fig. 4, this would be ≈ 1.15 cm/cm. If, in the actual experiment, the bullet penetrated the fabric, its residual velocity (measured by the experimental apparatus) is matched to the graph to identify the failure strain of the fabric. For a bullet exiting with velocity 200 m/s, the corresponding failure strain would be ≈ 0.9 cm/cm.

The preceding application of the algorithm involves actual experimental data input and provides as output a nominal failure strain. A second application of the algorithm would be for purposes of extrapolation. After a failure strain has been identified for a fabric, one can plug into the algorithm a different impact situation, such as additional layers of fabric or a different bullet shape or size. Based on that known failure strain, one can identify if the fabric will fail and, if so, at what residual velocity the bullet will exit.

It is important to note that since failure strain can be a function of strain rate, extrapolation of a failure strain to a different striking velocity may introduce errors.

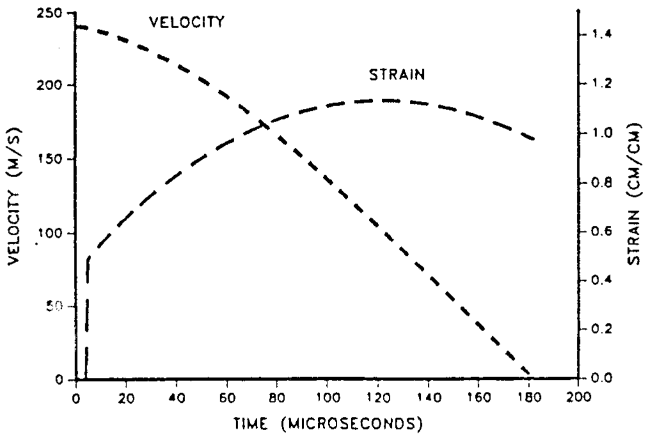

Fig. 4 Output of algorithm: allows matching residual velocity with a failure strain; data here are for layer 3 of 3 for a 0.357 caliber bullet striking at 240 m/s.

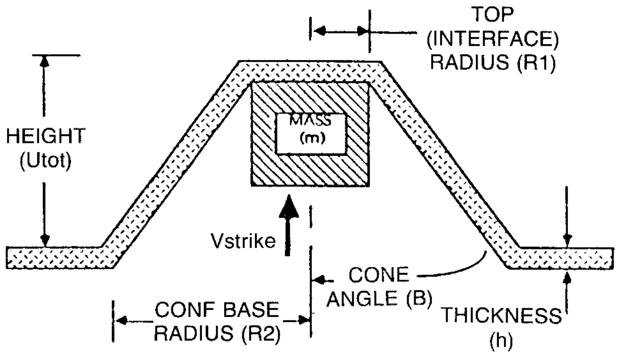

Fig. 5 Impact geometry.

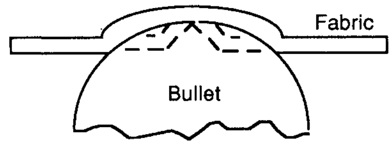

Fig. 6 Phase I of impact, when the transverse wave (dashed) cannot propagate; *R*1 has not been reached (exaggerated).

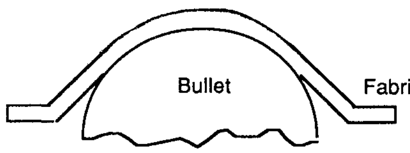

Fig. 7 Phase II of impact, when the transverse wave begins to propagate; *R*1 is determined by the radius at the separation point (exaggerated).

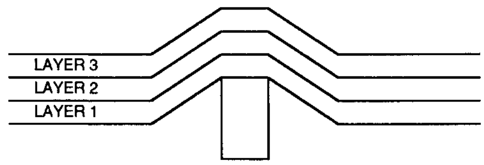

Fig. 8 Thick model: 0% compressibility.

III. Description of the Input Variables

Section II described how the dynamic problem is approximated by a series of static problems that are solved in succession to recreate the impact. At the heart of each static solution lies the conical shell theory. The input to the conical shell equations are the variables describing the geometry of the shell at that time instant and the material properties of the fabric. The output of the equations is simply the axial loading on the fabric, which as just described allows calculation of the velocities and positions to be used for the next time step.

Most of these input variables change throughout the impact and therefore must be recalculated at each time step. Again, since little is known of material properties at high strain rates, quantitative values for these variables require some derivation. The following explains these methods (see Fig. 5):

A. Geometry Variables

1. Base Radius (*R*2 in the Algorithm)

When the fabric is impacted, the fibers initially are pulled toward the impact point (in the plane of the fabric). Shortly thereafter they abruptly cease their radially inward motion and are forced to move transverse to the target, parallel to the bullet. The cone is formed. The radial position of where this transverse motion is just beginning (the front of the transverse wave) defines *R*2, the base radius. This position is found by approximating the velocity of the transverse wave front (*UBAR*), and multiplying this by the known time. This transverse wave speed is approximated in this study by studying data[3-6] for nylon yarns. From this it was possible to approximate *UBAR* as a function of striking velocity. The paper by Wilde et al.[5] provided a relationship between *UBAR* of a yarn and *UBAR* of a fabric resulting in a crude approximation for *UBAR* of the fabric as a function of striking velocity as

$$UBAR = 64 + (0.74 \cdot V_{\text{str}}) \tag{1}$$

A high-speed videocamera capable of 1 frame/83 μs was used to verify this approximation. By filming the impact against a gridded background, measurements of cone base radius (as well as other geometry variables) were obtained. Even at this film speed, only one or two pictures of an impact are captured, and thus the accuracy of the measured results is limited. It was found that Eq. (1) predicts (within a factor of four) the speed *UBAR*.

2. Top Radius (*R*1 in the Algorithm)

The top radius is defined by the contact circle where the bullet meets the fabric. A simplified model would assume that all bullets are flat cylinders and approximate *R*1 as one-half of the caliber of the bullet. Often though, projectiles have curved noses resulting in an *R*1 somewhat less than the radius of the bullet. This model is capable of accounting for these situations by combining the bullet nose shape, impact velocity, and transverse wave speed.

The identification of *R*1 is only possible when a transverse wave is able to form. This is prohibited during the first few microseconds of impact since the point of contact moves radially much quicker than the speed of the transverse wave that is trying to form (see Fig. 6). The time is incremented and the contact point is recalculated until the geometry dictates that the contact point does not move as rapidly as *UBAR*. The transverse wave conical shell is able to form, and the projectile will begin to decelerate (see Fig. 7). At this time the contact point defines radius *R*1, which will not change for the remainder of the impact.

3. Thickness (*h* in the Algorithm)

This is the virtual thickness of the material, obtained by dividing weight per area by mass density. In multiple layer targets, however, *h* takes on additional importance as the thickness determines the distance from the front of one layer to the front of the next. This dictates the time delay before a rear layer begins to contribute to the deceleration and, hence, affects the impact. The following analysis allows that situation to be modeled.

One could assume that every layer is affected as soon as the bullet strikes the first layer: in effect assuming that the fabric is incompressible in the transverse direction and that *N* layers acting identically will behave the same as one thick layer. Figure 8 shows this model.

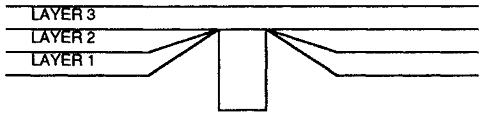

Fig. 9 Multiple layer model: 100% compressibility.

Instead of assuming that all layers are immediately affected, one can reason that the layers 2,..., N are not affected until a later time. A compressibility of 100% is assumed for the fabric. A layer N does not respond until the projectile has traveled a distance $(N-1) \times$ thickness. Figure 9 shows this model.

The impact is bounded by limits (0%, 100%) on the compressibility of the fabric. We have found that for targets of five layers or less, there is essentially no significant difference in the predictions from the two models.

4. Height (*Utot* in the Algorithm)

The height of the cone at any time is simply the distance the bullet has traveled. This is a direct output from the algorithm.

B. Material Properties

1. Strain (ϵ in the Algorithm)

This approach uses a maximum strain criteria as the definition of failure. In an axially loaded conical shell, the shearing strains are negligibly small, and the circumferential strain (in the hoop direction) is directly related to the meridional strain (along the shell's side) by Poisson's ratio. Therefore, the use of only the meridional strain suffices as a criterion for failure.

The strain in the conical shell is purely a function of the shell geometry. For a truncated conical shell loaded axially at the smaller end (top), the maximum strain occurs at that end and decreases linearly to some nonzero value at the base.[1]

2. Modulus (E in the Algorithm)

An estimate of the relationship between a material's modulus and the speed of sound through it is[7]

$$E = \rho C^2 \tag{2}$$

There exists another equation involving C and two other factors that are already measured or calculated during the algorithm, which provides the relationship among the transverse wave speed, the strain in the fiber, and the speed of sound as[4]

$$UBAR = C\{[\epsilon(1+\epsilon)]^{1/2} - \epsilon\} \tag{3}$$

The $UBAR$ is approximated as explained in the previous section. The strain implied in the equation is the strain in the material through which the transverse wave is moving (just ahead of the transverse wave), approximated here by the strain at the base of the cone, which is found at each time interval by the application of conical shell theory. From these equations, the modulus is found.

IV. Sample Calculation

An example, here for a 0.22 (curved nose, mass = 1.95 g) striking one layer of Kevlar 29 (density = 1.439 g/cm^3 and weight per area = 338 g/m^2) at 240 m/s, may help tie together the concepts. With several modifications, the equations in the algorithm found on page 265 of Ref. 1 can be used. Modify $R2_t$ in line 4 to $(R2_t - R1)$ and multiply the right-hand side of line 5 by $(R1/R2_t)$. Replace y_2/y_1 with $R2/R1$ in lines 5 and 8.

Determination of $R1$ is as follows. The bullet's nose shape is projected onto a screen, graphed, and subsequently approximated by computer as

$$y = 473.2r^5 - 49.26r^4 - 8.35r^3 + 2.34r^2 + 0.19r - 0.008 \tag{4}$$

where r is the radius at an axial distance y from the tip. The

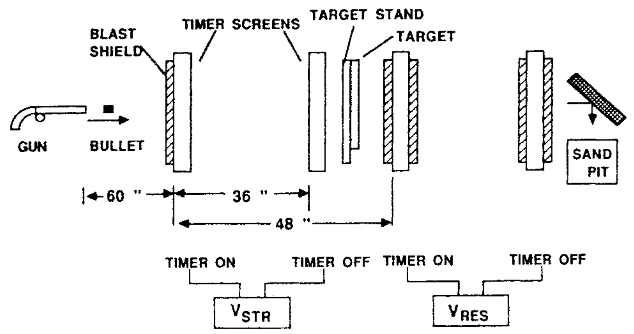

Fig. 10 Layout of apparatus.

velocity of the radial position of the contact point, as explained in Sec. IIIA, is found by

$$dr/dt = (dy/dt) / (dy/dr) \tag{5}$$

where dy/dt is the impact velocity and dy/dr is found from Eq. (4). This value equals $UBAR$ [found from Eq. (1) to be 241 m/s] when $r = 0.16$ cm, thus defining $R1$. The bullet has only traveled 0.065 cm in the 2.7 μs it takes to reach this position, and so the deflection of the fabric and deceleration of the bullet during this stage are assumed negligible and the algorithm can be started at this point. If the material fails at a strain of 1.147, the residual velocity should be approximately 217 m/s.

V. Experimental Apparatus

The experimental program is comprised of approximately 185 shots into Kevlar 29, 1500 denier plain weave fabric with a weight per area of 338 g/m^2. The 8-in.-square targets were composed of 1 to 5 layers. The data required are simply the striking velocity and the residual velocity of the bullet. Two chronograph systems ($\approx$ \$2000) were purchased to handle these duties (see Fig. 10). Both 0.22 (round nose, 1.95 g, $R1$ = 0.16 cm) and 0.357 (flat nose, 9.98 g, $R1$ = 0.453 cm) bullets were fired. Shots were grouped around three velocities: 240, 310, and 380 m/s. The 0.22 shots were fired from a 21-in. rifled barrel. All ammunition for these shots is commercially available in 29 grain bullet velocities called "CB short," "short," and "long." The 0.357 shots were fired from a Smith & Wesson Model 66 with a 6-in. barrel. In order to allow direct comparison of the two bullet types at the different speeds, the flat nosed 0.357 "semi-wad cutter" bullets were all hand-loaded to match the velocities of the three 0.22 shell loadings.

VI. Results

A. Model's Predictions vs Videotaped Results

In Figs. 11 and 12, we compare the model's predictions for base radius and cone height with the results measured from the video. It is seen that the predictions by the theory are close to the actual values for all shots. Throughout the range of striking velocities, target thicknesses, and bullet types, the model predicts the trends seen in the video results. Additionally, the elapsed time before penetration was estimated as either less than 83 μs or greater than 83 μs. In all five shots, the model agreed.

B. Discussion of Analytical and Experimental Results

1. Effect of Striking Velocity

At low-impact velocities, the fabric strains slightly and arrests the bullet. As impact speed increases, so does the strain, until eventually penetration can occur. Typical data are plotted in Fig. 13. The negative slope of the failure strain at the higher velocities indicates possibly a strain rate sensitivity of the target material. This emphasizes the point that extrapolations from one striking velocity to another can be erroneous.

2. Effect of Target Thickness

Not surprisingly, the model shows that in stopping a bullet, a thick target strains less than a thin target. An important part of this study, though, was a test of the assumption that the calculated failure strain for the fabric is independent of the number of layers in the target. An estimation of the failure strain of the fabric resulting from the tests on two layers of fabric should agree with an estimation resulting from tests on one layer of fabric. If these did not agree, then the model's versatility is severely limited. Figure 14 shows that the average failure strain for a given striking velocity shows no consistent dependency upon target thickness. This allows data for a shot through a single layer target to be used to predict penetration through a multiple layer target.

3. Effect of Projectile Type

Another aim of this model is the capability to account for different projectile shapes and sizes. If one were to obtain an average failure strain based on 0.22 penetration data, and then do the same using only 0.357 data, the results should match. Figure 15 shows this is true. This means that the data from one projectile can be used to predict penetration (at a similar velocity) of an entirely different bullet.

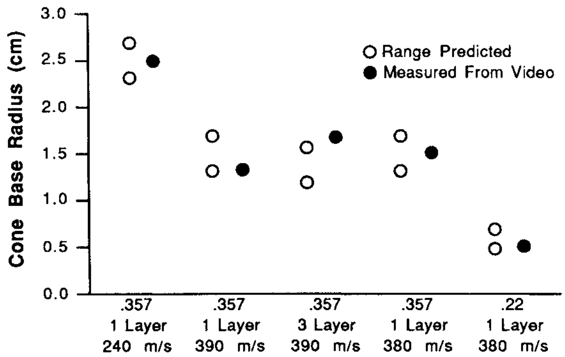

Fig. 11 Predicted vs measured results for cone base radius at time of penetration.

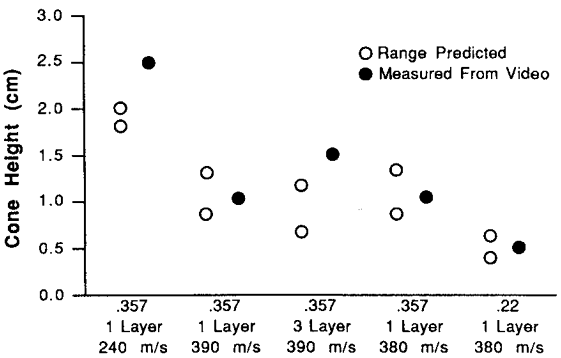

Fig. 12 Predicted vs measured results for cone height at time of penetration.

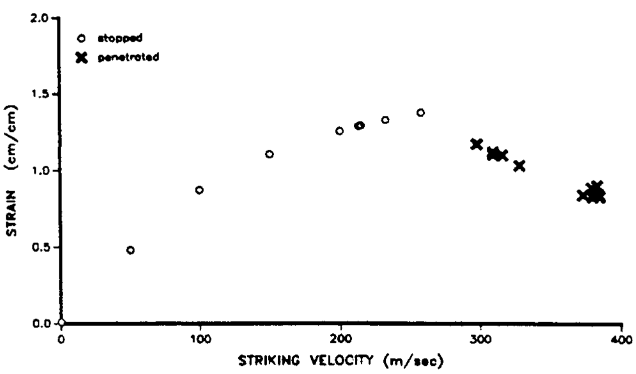

Fig. 13 Strain vs striking velocity; data are for 0.357 caliber bullets into two layers of fabric.

4. Validity of Predictions by the Model

Combination of the preceding results allows extrapolation of penetration data in one case to predict penetration in an en-

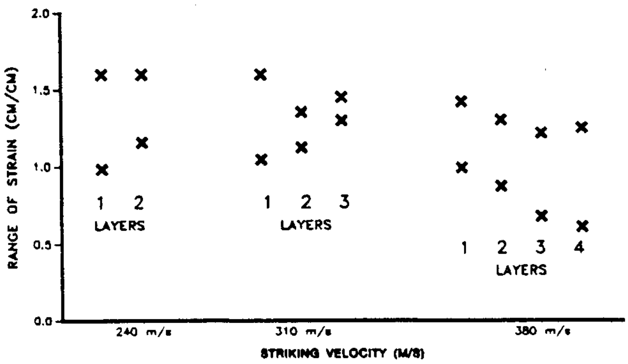

Fig. 14 Effect of target thickness on failure strain; data are for 0.22 caliber shots which penetrated.

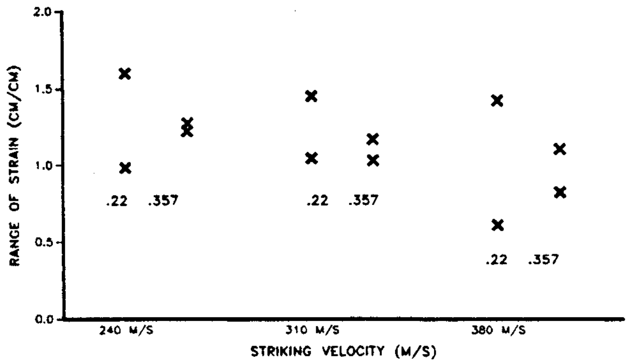

Fig. 15 Calculated range of failure strain; compare the prediction from 0.22 data with prediction from 0.357 data.

Table 1 Predicting 0.357 residual velocities based on 0.22 failure strains—data are for single-layer target

Striking velocity, m/s	Failure strain from 0.22 data	Predicted V_{res}	Actual V_{res}
240	1.147	217	209
310	1.186	283	290
380	1.033	362	367

Table 2 Predicting number of layers required to stop a 0.357 caliber bullet based on single-layer 0.22 failure strains

Striking velocity, m/s	Failure strain from 0.22 data	Predicted layers	Actual layers
240	1.147	3	2
310	1.186	4	3
380	1.033	6	5–6

Table 3 Predicting number of layers required to stop a 0.22 caliber bullet based on single-layer 0.22 failure strains

Striking velocity, m/s	Failure strain from 0.22 data	Predicted layers	Actual layers
240	1.147	5	3
310	1.186	6	4
380	1.033	10	5

tirely different impact situation. Based on penetration data for 0.22, the residual velocities for a 0.357 penetration impact were predicted to within several percent (see Table 1).

The use of penetration data to predict projectile arrest does not work as well (see Tables 2 and 3). Given a failure strain, the algorithm tends to predict higher number layers than are actually required for arrest. We believe a major factor for this discrepancy could be that projectile deformation is not presently included in the algorithm. Mushroom-shaped deformation of the bullet, causing increases in $R1$ of magnitudes up to 100%, was noted in all cases of arrest. This would tend to reduce the strain at the contact point (load is distributed over larger area) and result in lower predictions for layers required for arrest.

VII. Conclusions

A previously proposed model for ballistic impact into flexible materials has been thoroughly tested by an experimental program. It is seen to be a powerful model which, given penetration data from one impact condition, allows characterization of the material in a variety of totally different impact conditions. Required input is minimal and inexpensive. Salient results are as follows:

1) Behavior of a multiple-layer target can be modeled from the behavior of a single-layer target.

2) The model can accurately predict a penetrating impact. Differences are found when trying to predict the arrest of a projectile.

3) The model's predictions of the geometry of the deflected fabric during impact through a range of impact conditions are corroborated by high-speed videos.

4) Large deformations of the projectiles during impact occur and are currently not incorporated in the model. This is most likely responsible for errors in prediction of arrest. This topic should be of primary interest in any further work.

5) Accurate measurements of the striking and residual velocities are required. Additional work in the area would require velocity confidence intervals near 1–2 m/s.

Acknowledgments

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